Testing for Curves in a Binary Image

Y. C. Cheng

National Taipei University of Technology, Taipei 106, Taiwan
yccheng@ntut.edu.tw,
WWW home page: http://www.ntut.edu.tw/~yccheng/

Abstract. Curve detection is viewed as a process of hypothesis generation and hypothesis testing. Of the two, hypothesis generation has received much attention and many sophisticated post-processing strategies are published in the literature. In this work, the emphasis is shifted to the development of an efficient and effective hypothesis testing strategy to relieve the hypothesis generation from sophisticated computations. The proposed method recasts edge pixels in a binary image into a one-parameter system derived from the hypothesis. The recasting process creates a histogram, which contains a single dominant peak if and only if the hypothesis under testing contains a significant number of edge pixels. Experiments with circle testing show that the proposed strategy outperforms the global threshold.

1 Introduction

Curve detection is an elementary step in extracting shape information from a binary image. In principle, curve detection can be viewed as a process of hypothesis generation, which finds instances of curves that are likely to be in the image, and hypothesis testing, which verifies that these instances indeed exist in the image. While the strategy for hypothesis generation varies for different types of curve detectors, (e.g., evidence accumulation in Hough transforms [1] and consensus set computation in RANSAC methods [2,3]), hypothesis testing is the common step within a curve detection method to finally decide which curves to accept.

Given a set of hypotheses, an ideal hypothesis testing strategy is one that keeps all true positives and eliminates all false positives. Note that the definition depends on hypothesis generation. In one extreme, if a strategy generates all true positives and no false positives, then a trivial testing strategy that accepts all hypotheses by default is ideal. In the other extreme, if both true positives and false positives are generated, a testing strategy is what finally decides the missing percentage and the false positive percentage within the limits set by hypothesis generation. Furthermore, note that under our definition of hypothesis generation and hypothesis testing, post-processing strategies applied in the parameter space (e.g., peak sharpening and filtering in Hough space [4]) are considered a part of hypothesis generation. For example, the peak sharpening strategy of Gerig and Klein [5] is accomplished by doing the Hough transform twice: the first
pass does the standard accumulation process and the second pass assigns each edge pixel to the highest-counting bin that is intersected by its hypersurface in the parameter space. The result is strongly sharpened peaks at the cost of twice the computing time and memory space of a standard Hough transform. Survey papers on the curve detection literature reveal that most of the effort so far has focused on creating the better hypothesis generation strategies [4, 6]. This partly explains why the simple global threshold is still the most commonly used hypothesis testing strategy. However, as we shall show, the simple global threshold should not be used with detectors with a less sophisticated hypothesis generation strategy.

In this paper, we explore the possibility of developing effective hypothesis testing strategies, in the hope to avoid extra computing effort in the hypothesis generation. We propose a general strategy that tests hypotheses as follows. To test an instance of curve under detection, a one-parameter system of curves, one that the instance is a member of, is constructed. A transformation based on the one-parameter system is created and applied to edge pixels in the image; the result is a one-dimensional distribution-counting histogram of the edge pixels in the image against a range of specific members of the one-parameter system. The instance passes the test if and only if the histogram contains a single dominant peak.

2 Hypothesis testing

Let \( h \) be an instance of a curve (e.g., a line, a circle or an ellipse) found by a hypothesis generation strategy such as Hough transform or RANSAC. The instance \( h \) can be represented as a member of a one-parameter system of curves:

\[
s(\lambda) = (1 - \lambda)s_1 + \lambda s_2 = 0, \lambda \in \mathbb{R},
\]

where \( s_1 = 0 \) and \( s_2 = 0 \) are two distinct curves of the same type as \( h \) and intersect \( h \) at one, two, or four points, respectively, if \( h \) is a line, a circle, or an ellipse, respectively.

Edge pixels \((x, y)\) in an image can be mapped into a one-dimensional parameter space with the function \( \lambda_{s_1, s_2} : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \) derived from Equation (1):

\[
\lambda_{s_1, s_2}(x, y) = \frac{s_1(x, y)}{s_1(x, y) - s_2(x, y)}.
\]

It is easy to see that edge pixels on the instance \( h \) are mapped into the same value by the function \( \lambda_{s_1, s_2} \). Computationally, the cumulative result of the mapping is represented by a histogram of number of edge pixels against a range of \( \lambda \) values.

Let \( j \) be a curve that is not a member of the one-parameter system constructed from \( h \). Edge pixels on \( j \) is uniformly distributed into the histogram since \( j \) is intersected by each of the members in the one parameter system in at most \( \text{deg}(j) \cdot \text{deg}(h) \) points, where \( \text{deg} \) is the degree of a curve. For outliers categorized as uniformly distributed noise with noise level \( \gamma \), the number of noise
edge pixels accumulated in a bin is a binomial random variable \([3]\) with an expected value of \(\gamma B(\lambda)\), where \(B(\lambda)\) is the number of pixels on the curve \(s(\lambda)\). Therefore, we have

**Theorem 1.** The histogram constructed from the hypothesis contains a single dominant peak if and only if the hypothesis contains a significant number of edge pixels.

In the histogram obtained by classifying the edge pixels into curves of a one-parameter system constructed from the hypothesis, the presence of the single dominant peak indicates that the hypothesis is statistically more likely to be a true positive than other curves. This property allows us to devise an adaptive threshold. In contrast, when a hypothesis is accepted by a global threshold, there is no statistical support to claim that it is more likely to be a true positive than other curves.

The proposed hypothesis testing strategy involves the following steps. (1) Determine the base curves \(s_1 = 0\) and \(s_2 = 0\) from the instance \(h\). (2) Accumulate the histogram by applying the function \(\lambda_{s_1,s_2}(x,y)\) to edge pixels in the image. (3) Detect the existence of a single dominant peak in the histogram. Next, the process is illustrated with circle testing.

### 2.1 Circle testing

**Construction of base curves.** Let \(h\) be a circle with parameter \((x_0, y_0, r_0)\) found by a hypothesis generation strategy. Choosing base curves to have parameters \((x_0 - r_0, y_0, \sqrt{2}r_0)\) and \((x_0 + r_0, y_0, \sqrt{2}r_0)\), the one-parameter system (coaxal system \([8]\)) is defined

\[
(1-\lambda)((x-x_0+r_0)^2+(y-y_0)^2-2r_0^2)+\lambda((x-x_0-r_0)^2+(y-y_0)^2-2r_0^2) = 0. \tag{3}
\]

The construction is illustrated in Figure 1. Note that \(h, s_1\) and \(s_2\) are the members at \(\lambda = 1/2, 0\) and 1, respectively.

**Computing histogram.** The transform defined from coaxal system in (3) is

\[
\lambda_{s_1,s_2}(x,y) = \frac{(x-x_0 + r_0)^2 + (y-y_0)^2 - 2r_0^2}{4r_0(x-x_0)}, \tag{4}
\]

edge pixels in the image is mapped into a histogram covering the range of \(\lambda\) value in \([0, 1]\).

**Detecting the single dominant peak in histogram.** Based on Theorem 1, an adaptive threshold for hypothesis testing can be devised. First, the sample mean \(\bar{X}\) and sample variance \(S^2\) \([9]\) of the histogram except the peak are calculated. The hypothesis is accepted if and only if pixel count in the histogram is greater than \(\bar{X} + wS\). A large \(w\) ensures a low false positive rate at the cost of a higher missing rate. A small \(w\) ensures a low missing rate at the cost of a high false positive rate.
Once tuned, the value of $w$ needs not be re-tuned when the image complexity changes. Since if the image complexity is high (respectively, low), the sample mean and sample variance will be correspondingly large (respectively, small), which leads to a high (respectively, low) threshold. In contrast, a fixed threshold is sensitive to the image complexity.

3 Experiments

In this section, the performance of the proposed hypothesis testing strategy is compared with the global threshold in the context of circle detection. The standard Hough transform for circle is used for hypothesis generation. The test images are synthetic images containing a number of circles of varying degrees of completeness in the presence of uniform pepper-and-salt noise. The set of hypotheses generated is denoted by $Q$. The performance of the testing strategies are measured in terms of the missing rates ($m$) and the false positive rates ($f$), where

$$m = \frac{\text{number of true curves in } Q \ - \ \text{number of true curves accepted}}{\text{number of true curves in } Q},$$

and

$$f = \frac{\text{number of false positives accepted}}{\text{number of curves accepted}}.$$

To correct the preference to large circles, which have more edge pixels than small circles, both the global threshold and the histogram of the proposed method are radius-scaled. Two threshold values for each testing strategy are recorded, the maximum threshold $t_1$ that all of $Q$ are accepted and the minimum threshold
that all of $Q$ are rejected. Only threshold values within $[t_l, t_r]$ are considered. Further, to facilitate comparison, the performance figures $m$ and $f$ at the threshold value of $t$ are plotted against $t_{\text{normalized}}$:

$$t_{\text{normalized}} = \frac{t - t_l}{t_r - t_l}.$$  

In particular, we are interested in the range of threshold values for which $m = 0$ and $f = 0$. The range is called the ideal threshold range of operation.

The performance plots for input images at noise levels of 0%, 3% and 5% are shown in Figures 2, 3, and 4. Except in the noise-free plot (Figure 2), the proposed testing strategy consistently outperforms the global threshold in the false positive rates. Moreover, as the image becomes increasingly noisy, the ideal range of operation for the global threshold narrows and nullifies eventually (Figure 4). A null ideal range of operation means that no matter how the threshold value is set, either some false positives are accepted or some true positives are rejected. In contrast, the proposed strategy has a relatively wide and stable ideal range of operation regardless of noise.

![Graph](image_url)

**Fig. 2.** False positive rates and missing rates against normalized threshold (noise free)
Fig. 3. False positive rates and missing rates against normalized threshold (3% uniform noise)

4 Concluding remarks

A hypothesis testing strategy for use in curve detection is proposed. The strategy recasts edge pixels in an image into a one-parameter system. Computationally, the proposed strategy is as efficient as the global threshold. Experiments on circle testing show that it not only outperforms the global threshold strategy, but also exhibits a wider and relatively stable ideal range of operation. It can be used in conjunction with standard Hough transforms [1], randomized Hough transforms [7], and RANSAC based methods [2, 3, 10], which are computationally more efficient and more applicable in resource stringent situations. The implementation of testers for lines and ellipses is not difficult and is underway.

References

Fig. 4. False positive rates and missing rates against normalized threshold (5% uniform noise)