程序控制
閉環路系統頻率應答分析
控制系統之開環頻率應答

\[ G_{OL}(i\omega) = G_c \cdot G_v \cdot G_p \cdot G_m(i\omega) \]
閉環路系統之頻率應答

\[ G_{CL}(s) = \frac{Y(s)}{Y_{sp}(s)} = \frac{G_c G_p}{1 + G_c G_p} = \frac{G_{OL}}{1 + G_{OL}} \quad (G_{OL}(s) = G_c G_p(s)) \]

- \( G_{CL}(s) \) 的頻率應答即為閉環系統之頻率應答

\[ G_{CL}(i\omega) = u + iv = M e^{i\theta} \]

\[ |G_{CL}(i\omega)| = \text{閉環系統之振幅比} = M \]

\[ \angle G_{CL}(i\omega) = \text{閉環系統之相角} = \theta \]
閉環路系統之頻率應答可由開環頻率應答求得

令 \[ G_{OL}(i\omega) = \beta e^{i\phi} \quad \text{和} \quad G_{CL}(i\omega) = M e^{i\theta} \]

而 \[ G_{CL}(i\omega) = \frac{G_{OL}(i\omega)}{1 + G_{OL}(i\omega)} = \frac{\beta e^{i\phi}}{1 + \beta e^{i\phi}} = \frac{\beta (\cos \phi + i \sin \phi)}{1 + \beta (\cos \phi + i \sin \phi)} \]

故 \[ |G_{CL}(i\omega)| = M = \left| \frac{\beta e^{i\phi}}{1 + \beta e^{i\phi}} \right| = \frac{\beta}{\sqrt{(1 + \beta \cos \phi)^2 + (\beta \sin \phi)^2}} \]

\[ \angle G_{CL}(i\omega) = \theta = \phi - \tan^{-1}\left( \frac{\beta \sin \phi}{1 + \beta \cos \phi} \right) \]

- 若 \( \beta \) 及 \( \phi \) 為已知，則 \( M \) 及 \( \theta \) 即可求得
Example

$G_{OL}(s) = \frac{5}{(s+1)(2s+1)}$

Bode plot for $G_{OL}$
Bode plot for $G_{CL}$

Set-point step response
Example: 控制器設計之衝突

・ 閉環路動態行為

\[ Y = \frac{G_c G_p}{1 + G_c G_p} Y_{sp} + \frac{G_d}{1 + G_c G_p} L - \frac{G_c G_p}{1 + G_c G_p} N \]
若設計控制器使 $|G_cG_p| \gg 1$ (在某頻率範圍內)，則

$$\frac{Y}{Y_{sp}} = \frac{G_cG_p}{1+G_cG_p} \approx 1 \implies Y \approx Y_{sp} \quad \text{(Good set-point tracking)}$$

$$\frac{Y}{L} = \frac{G_d}{1+G_cG_p} \approx 0 \implies Y \approx 0 \quad \text{(Good disturbance rejection)}$$

然而

$$\frac{Y}{N} = -\frac{G_cG_p}{1+G_cG_p} \approx -1 \implies Y \approx -N \quad \text{(Sensitive to noise)}$$
閉環路系統之穩定性

一般穩定性準則

- Characteristic equation

\[ 1 + G_C G_p(s) = 1 + G_{OL}(s) = 0 \]

- Stability requirement

All roots of the characteristic equation lie in LHP Plane

No root lies here
Bode Criterion for Stability

• Definitions
  – Critical frequency (phase crossover frequency) $\omega_c$
    \[ \angle G_{OL}(i\omega_c) = -180^\circ \]
  – Gain crossover frequency $\omega_g$
    \[ |G_{OL}(i\omega_g)| = 1 \]

• Bode Stability Criterion
  – Consider a stable open-loop transfer function $G_{OL}(s)$. For the closed-loop system to be stable, the amplitude ratio of $G_{OL}(s)$ must be less than one when its phase angle is $-180^\circ$
    \[ 1 + G_{OL}(s) = 0 \Rightarrow G_{OL}(s) = -1 \]
    \[ |G_{OL}(i\omega)| = 1, \quad \angle G_{OL}(i\omega) = -180^\circ \]
  – If $|G_{OL}(i\omega_c)| < 1$, the system is stable
  – If $|G_{OL}(i\omega_c)| > 1$, the system is unstable
Bode stability criterion 特性

• It provides a necessary and sufficient condition for closed-loop stability based on the properties of the open-loop transfer function.

• Unlike the Routh stability criterion, the Bode stability criterion is applicable to systems that contain time delays.

\[ \angle G_{OL}(i\omega_c) = -180^\circ \]
\[ |G_{OL}(i\omega_c)| = 1 \]
Example

- Bode plots for $G_{OL}(s) = 2Kc/(0.5s+1)^3$

$$|G_{OL}(j\omega)|$$

$$\angle G_{OL}(j\omega)$$

$G_{OL} = K_c G_p$

$$K_{c,u} = \frac{1}{|G_p(i\omega_c)|}$$
Nyquist Criterion for Stability

• The Nyquist stability criterion is similar to the Bode criterion in that it determines closed-loop stability from the open-loop frequency response characteristics

• **Nyquist Stability Criterion**
  
  – Consider a *stable* open-loop transfer function $G_{OL}(s)$. The closed-loop system is stable if and only if the Nyquist plot (polar plot) for $G_{OL}(s)$ doesn’t encircle the (-1,0) point
Example

\[ G_{OL}(s) = G_c G_p(s) = K_c \frac{4e^{-s}}{5s + 1} \]

若 \( K_c = 1.5K_{c,u} \)，則此閉環路系統是否穩定？

Nyquist plot for \( K_c = 1.5K_{c,u} = 6.38 \)

Encircle (-1,0)

\[ K_{c,u} = \frac{1}{|G_p(i\omega_c)|} = \frac{1}{0.235} = 4.25 \]

The closed-loop system is **unstable**
Measure of Relative Stability

• Recall
  – Critical frequency (phase crossover frequency) \( w_c \)
    \[ \angle G_{OL}(iw_c) = -180^\circ \]
  – Gain crossover frequency \( w_g \)
    \[ |G_{OL}(iw_g)| = 1 \]

• Gain margin (GM) is defined as
  \[ \text{GM} \triangleq \frac{1}{|G_{OL}(i\omega_c)|} \]

• Phase margin (PM) is defined as
  \[ \text{PM} \triangleq \angle G_{OL}(i\omega_g) + 180 \]
Gain and Phase Margins in Bode Plots

\[ |G_{OL}(iw)| \]

\[ \frac{1}{GM} \]

\[ \angle G_{OL}(i\omega) = \phi_g \]

\[ |G_{OL}(i\omega_c)| = \frac{1}{GM} \]
Gain and Phase Margins on Nyquist Plot

\[ \omega = \omega_c \]

\[ \omega = \omega_g \]
Gain and Phase Margins

• **Relation to stability**
  - **Stable:** GM > 1; PM > 0
  - **Critically stable:** GM = 1; PM = 0
  - **Unstable:** GM < 1; PM < 0

• In general, large values of GM and PM correspond to sluggish closed-loop responses, while smaller values result in less sluggish, more oscillatory responses.

• **In general, a well-tuned controller should have a gain margin between 2 and 4** and a **phase margin between 30° and 60°**
GM 與 PM 之意義

- 一個控制系統之 GM 值代表，如果將此控制系統之控制器的 Kc 值放大 GM 倍，則此控制系統將會變成臨界穩定。

- GM 值的另外一個涵意就是如果用來設計控制器的程序模式與真實的程序模式除了程序增益以外，其餘動態部份完全一樣，則當真實程序的程序增益是用來設計控制系統之程序增益的 GM 倍時，控制系統臨界穩定；換句話說，如果設計的控制系統之增益邊距=GM*，則控制系統可以忍受真實程序之 Kp 比設計時所用之 Kp 大 GM*倍的錯誤，只要錯誤不要超過 GM*倍的範圍，控制系統仍然穩定。

- PM 值的另外一個涵意就是如果用來設計控制器的程序模式與真實的程序模式除了遲延以外，其餘動態部份完全一樣，則當真實程序的遻延比用來設計控制系統之遻延大 \( \frac{PM}{\omega_c} \cdot \frac{\pi}{180^\circ} \) 時，控制系統臨界穩定 (\( \omega_c \) 為 AR=1 時之頻率，且 PM 的單位是度/度量不是徑度量，如果 PM 採用徑度量，則公式為 PM/\( \omega_c \)。)
Example

• An open-loop transfer function

\[ G_{OL}(s) = G_c G_v G_p G_m(s) = G_c \frac{2e^{-s}}{5s+1} \]

determine the gain and phase margins for the following two PID controllers from their bode plots

<table>
<thead>
<tr>
<th>Controller Settings</th>
<th>( K_c )</th>
<th>( \tau_I ) (min)</th>
<th>( \tau_D ) (min)</th>
</tr>
</thead>
</table>
• Bode plot of $G_{OL}(s)$
The Tyreus-Luyben controller settings are more conservative owing to the larger gain and phase margins.

<table>
<thead>
<tr>
<th>Controller</th>
<th>$GM$</th>
<th>$PM$</th>
<th>$\omega_c$ (rad/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ziegler-Nichols</td>
<td>1.6</td>
<td>40°</td>
<td>2.29</td>
</tr>
<tr>
<td>Tyreus-Luyben</td>
<td>1.8</td>
<td>76°</td>
<td>2.51</td>
</tr>
</tbody>
</table>
Homework #6

1. 某一熱傳程序，其溫度 $T$ 與進料流量 $Q$ 之轉移函數為

$$\frac{T(s)}{Q(s)} = \frac{3(1-s)}{s(2s+1)}$$

若流量 $Q$ 發生正弦 $(\sin)$ 變化，振幅為 2，週期為 0.5，則達穩態時溫度變化之振幅為何？

2. 某程序之轉移函數為 $G_p(s) = \frac{3}{8s+1} e^{-4s}$ (忽略控制閥與量測器)，

請利用頻率應答分析及連續還圈法設計PID控制器。
（提示：$\omega$ 之範圍：0.3 ~ 0.6）
3. 如下圖之控制系統，

\[ G_c(s) = 2 \left( 1 + \frac{1}{5s} \right) \quad G_p(s) = \frac{2.5}{8s + 1} e^{-1.6s} \]

(1) 請計算此系統之增益邊距 (GM) 與相位邊距 (PM)

(提示：\( \omega \) 之範圍：0.5 ~ 1.0)

(2) 若控制器增益改變為 4，則此控制系統是否穩定？