1. Suppose that $X$ is a random variable with mean and variance both equal to 20. What can be said about $P\{0 < X < 40\}$?

2. One has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

3. A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. If the mean lifetime of this type of component is 100 hours and its standard deviation is 30 hours, how many of these components must be in stock so that the probability that the system is in continual operation for the next 2000 hours is at least .95?

4. For an arbitrary random variable $X$, use the Chebyshev inequality to show that the probability that $X$ is more than $k$ standard deviations from its expected value $E[X]$ satisfies

$$P[|X - E[X]| \geq k\sigma] \leq \frac{1}{k^2}$$

For a Gaussian random variable $Y$, use the $\Phi(\cdot)$ function to calculate the probability that $Y$ is more than $k$ standard deviations from its expected value $E[Y]$. Compute the result to the upper bound based on the Chebyshev inequality.

5. An experimental trial produces random variables $X_1$ and $X_2$ with correlation $r = E[X_1X_2]$. To estimate $r$, we perform $n$ independent trials and form the estimate

$$\hat{r}_n = \frac{1}{n} \sum_{i=1}^{n} X_1(i)X_2(i)$$

where $X_1(i)$ and $X_2(i)$ are samples of $X_1$ and $X_2$ on trial $i$. Show that if Var$[X_1X_2]$ is finite, then $\hat{r}_1, \hat{r}_2, \ldots$ is an unbiased, consistent sequence of estimates of $r$.

6. Let $X_1, X_2, \ldots$, denote an iid sequence of random variable, each with expected value 75 and standard deviation 15.

(a) How many samples $n$ do we need to guarantee that the sample mean $M_n(X)$ is between 74 and 76 with probability 0.99?

(b) If each $X_i$ has a Gaussian distribution, how many samples $n'$ would we need to guarantee $M_{n'}(X)$ is between 74 and 76 with probability 0.99?
7. When we perform an experiment, event $A$ occurs with probability $P[A] = 0.01$. In this problem, we estimate $P[A]$ using $\hat{P}_n(A)$, the relative frequency of $A$ over $n$ independent trials.

(a) How many trials $n$ are needed so that the interval estimate

$$\hat{P}_n(A) - 0.001 < P[A] < \hat{P}_n(A) + 0.001$$

has confidence coefficient $1 - \alpha = 0.99$?

(b) How many trials $n$ are needed so that the probability $\hat{P}_n(A)$ differs from $P[A]$ by more than 0.1% is less than 0.01?