Computer Science and Information Engineering
Numerical Methods

Examination I

09:10 – 10:10 am
November 17, 200

This is an Open Book examination. Only notes or references belonging to you are acceptable. The examination consists of FOUR problems of equal weight.

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Problem 1: [50 points]

Show how one can rearrange the expression $e^x - e^{-x}$ so as to avoid cancellation when $x$ is close to 0.
Problem 2: [50 points]
Let $A$ be a matrix given by
\[
A = \begin{bmatrix}
-1 & 2 & 1 \\
-2 & 3 & 1 \\
-1 & 2 & 3
\end{bmatrix}.
\]

(a) Using Gaussian elimination with partial pivoting, obtain the LU decomposition of $PA$, where $P$ is a permutation matrix. That is, $PA = LU$. Show all steps.

(b) What is the determinant of $A^{-1}$, $\det(A^{-1})$?
Problem 3: [50 points]

Suppose

\[ f = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}. \]

(a) Consider the linear system \( Ax = f \) where \( A \) is given in Problem 2. Use the decomposition in Problem 2 to obtain \( x \) in \( LUx = Pf \).

(b) Obtain \( \| A \|_\infty \) and \( \| A^{-1} \|_\infty \).
Problem 4: [50 points]

True or false. Please verify your every answer shortly.

(a) If $\|A\|_1 \leq 1$, then $|a_{ij}| \leq 1$, where $A$ is a matrix.

(b) If $\|A\|_1 \leq \|B\|_1$, then $|a_{ij}| \leq |b_{ij}|$, where $A$ and $B$ are matrices.

(c) For any vector $x$ of degree $n$, $\frac{1}{n} \|x\|_1 \leq \|x\|_\infty \leq \|x\|_1$.

(d) Consider the two equations in $x$, and $y$

\[
\begin{align*}
x + \beta y &= 1 \\
\beta x + y &= 0;
\end{align*}
\]

$\beta > 0$.

The condition number of the problem of determining $y(\beta)$ alone, is given by

\[
\frac{2\beta^2}{(1 - \beta^2)}.
\]

(e) Let $z = (0.3)_{10}$. Then, $z$ has a nonterminating binary representation.