1. (20pts) Use a Taylor polynomial about $\frac{\pi}{4}$ to approximate $\cos 42^\circ$ to an accuracy of $10^{-6}$.

2. (20pts) Prove that the Maclaurin series of a function has only even powers if and only if the function is even.

3. (20pts) The number $p^*$ approximates $p = 2.7182$ to four significant digits. Please find the largest interval in which $p^*$ can lie.

4. (20pts) Suppose two points $(x_0, y_0)$ and $(x_1, y_1)$ are on a straight line. Two formulas are available to find the $x$-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \quad \text{and} \quad x = x_0 - \frac{(x_1 - x_0)y_0}{y_1 - y_0}.$$  

(a) Show that both formulas are algebraically correct.

(b) Using the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ and three-digit rounding arithmetic, compute the $x$-intercept both ways. Which method is better and why?

5. (20pts) Show how to avoid cancellation error for the following expressions:

(a) $\frac{1}{1+\sqrt{1+x}} + \frac{1}{1-\sqrt{1+x}}$, $x > 0$.

(b) $\frac{1-\cos x}{x}$, $x > 0$.

6. (20pts) Execute the following MATLAB program to compute $e^x$ using first $n$ terms of the expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots.$$  

```matlab
n = 50;
clf;
for x = [-10 10],
    term = 1; sum = 1;
    result = exp(x);
    for k=1:n,
        term = x .* term/k;
        sum = sum + term;
    end
    fprintf('x = %g, sum = %g, result = %g
', x, sum, result);
end
```
error(k) = abs(result-sum);
end;
relerr = error ./ result;
fprintf('x=%3d \ t exp(x)=%10d \ t err=%10d \ t relerr=%10d\n',...
x,result,error(n),relerr(n));
semilogy(x:1:n, relerr); hold on;
end;

(a) Why is the relative error larger for negative value of \(x\), e.g., for \(x = -10\)?
(b) Show that the relative error in computing \(f^{-1}\) is similar to the relative error in computing \(f(x)\). (Assume that \(\hat{f}(x) = f(x)(1 + \epsilon)\), where \(\epsilon\) is the relative error.)
(c) Suggest an alternative way to compute \(e^x\) for large negative values of \(x\)? (Hint: the relative error is smaller for \(x = 10\).)
(d) Verify that your method works by modifying the above code, and plot \(\text{relerr}\) versus \(k\). (The only change in your plot will be for \(x = -10\).)