1. (10 pts) Find a regular expression for the set \( \{ a^n b^m : n \geq 3, m \text{ is even} \} \).

   Answer:
   \[ r = aaaa^* (bb)^*. \]

2. (10 pts) Give a simple verbal description of the language \( L((aa)^* b(aa)^* + a(aa)^* ba(aa)^*) \).

   Answer: All strings are of the form \( w_1 b w_2 \), where \( w_1 \) and \( w_2 \) are composed of an even number of \( a \)'s, or \( w_1 \) and \( w_2 \) consists of an odd number of \( a \)'s.

3. (20 pts) Give regular expressions for the following languages.
   (a) \( L = \{ w : n_a(w) \text{ mod } 5 > 0 \} \) on \( \Sigma = \{ a, b \} \).
   (b) \( L = \{ w : n_a(w) \text{ mod } 5 > 0 \} \) on \( \Sigma = \{ a, b, c \} \).

   Answer:
   (a) \[ r = (b^* ab^* + b^* ab^* ab^* + b^* ab^* ab^* ab^*) (b^* ab^* ab^* ab^*)^*. \]
   (b) \[ r = ((b+c)^* a(b+c)^* + (b+c)^* a(b+c)^* a(b+c)^* + (b+c)^* a(b+c)^* a(b+c)^* + (b+c)^* a(b+c)^* a(b+c)^* (b+c)^* a(b+c)^* a(b+c)^* a(b+c)^*)^*. \]

4. (10 pts) Find a df a that accepts \( L(a^* bb) \cup L(ab^* ba) \).

   Answer:

5. (10 pts) Find the regular expression for the language accepted by the following automata.
Answer:

\[ r = b^*aa^*ab^*a. \]

6. (10 pts) Construct a right-linear grammar for the language \( L((aab^*ab)^*) \).

Answer:

\[ G = (V, T, S, P) \]

\[ S \rightarrow aaA | \lambda \]

\[ A \rightarrow bA | abS \]

7. (10 pts) Show that any regular grammar \( G \) for which \( L(G) \neq \emptyset \) must have at least one production of the form

\[ A \rightarrow x \]

where \( A \in V \) and \( x \in T^* \).

Answer: If there is no such rule, a sentence can never be derived.

8. (20 pts) Find regular grammars for the following languages on \( \{a, b\} \).

(a) \( L = \{w : (n_a(w) - n_b(w)) \mod 3 \neq 1\} \).

(b) \( L = \{w : |n_a(w) - n_b(w)| \text{ is odd}\} \).

Answer:

(a)

\[ S \rightarrow A | AaAaA | AAbAb. \]

\[ A \rightarrow aAb | bAa | \lambda. \]

(b)

\[ S \rightarrow AaA | AbA. \]

\[ A \rightarrow aAb | bAa | \lambda. \]

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