Abstract — Orthogonal subspace projection (OSP) has been successfully applied in hyperspectral image processing. In order for the OSP to be effective, the number of bands must be no less than that of endmembers to be classified, i.e., the number of equations have to be more than or equal to that of unknowns. This is known as Band Number Constraint. Such constraint is not an issue for hyperspectral images since they generally have hundreds of bands. However, this may not be true for multispectral images where the number of signatures to be classified might be greater than the number of bands such as 3-band SPOT XS images. The generalized version of OSP has been developed, called generalized OSP (GOSP) to relax this constraint in such a manner that the OSP can be extended to multispectral image processing in an unsupervised fashion. The idea of the GOSP is to create a new set of additional bands that are generated nonlinearly from original multispectral bands prior to the OSP classification. Since those additional bands are generated nonlinearly, for linear mixture model, this also introduces error. In this paper, we analyze the error resulting from band generation process with each nonlinear function used for generating additional bands. And then we further propose an approach to select a set of nonlinear functions for GOSP which will yield better classification results.

I. INTRODUCTION

With the improvement of remote sensing techniques, hyperspectral imagery now uses hundreds of contiguous bands to acquire data, which can discriminate and quantify materials more effectively in much narrower ranges than those used by multispectral imager which uses only tens of discrete bands. But the data volumes are substantially increased and general image processing techniques may not be adequate for such large dimensionality. In a widely used approach [1], Harsanyi and Chang developed an orthogonal subspace projection (OSP)-based classifier which not only could reduce dimensionality, but also showed success in hyperspectral image classification [2]-[5]. It was further shown to be an optimal linear classifier in terms of least squares error [5] and equivalent to the maximum likelihood classifier in [6]-[7] with Gaussian noise assumption.

In order for the OSP to be effective, the band number must be no less than the number of signatures to be classified, i.e., the number of equations has to be more than number of unknown variables. This constraint, referred to as band number constraint (BNC) requires that the data should have sufficient dimensions to perform least square unmixing. In this case, each individual signature can be classified in a separate dimension for discrimination. The phenomenon of the BNC was first witnessed when OSP performed poorly in discriminating four signatures using 3-band SPOT data, particularly those with similar spectra [8].

One previous proposed approach reduced the number of signatures to be classified so that the OSP could be applied to multispectral imagery. It requires all possible combinations of signatures be examined, and then a best classification result could be selected by visual inspection for final classification. Previously, we proposed Generalization Orthogonal Subspace Projection (GOSP) [9]. Instead of reducing the number of signatures as proposed in [8], we expand the original bands by creating a number of new bands. These new bands are generated nonlinearly in an attempt to capture nonlinear correlation between the original bands and can be used to improve classification performance. It is worth noting that linearly generated bands do not provide any useful information in linear mixture analysis. Incorporating these new nonlinearly correlated bands into original bands increases the number of bands to meet the BNC. Although the experiments show acceptable results, theoretically each additional image generated by nonlinear function will introduce an error into linear mixture equations. In this paper, we will analyze the errors resulting from the selection of nonlinear functions and also compare the classification results.

II. LINEAR MIXTURE MODEL

Linear spectral mixing analysis is a widely used approach in remotely sensing image processing to uncover endmember residents in a pixel area [1-4]. Let \( \mathbf{r} \) be an \( L \times 1 \) column vector and denote the spectral signature of a pixel vector in a multispectral or hyperspectral image with dimension \( L \), i.e., the number of spectral bands. Assume that \( \mathbf{M} \) is an \( L \times p \) matrix.
The idea of BGP [9] arises from the fact that a second-order random process is generally specified by its first-order and second-order statistics. If we view the original bands as the first-order statistical images, we can generate a set of second-order statistical bands by capturing correlation between bands. Let \( \{ B_i \}_{i=1}^p \) be the set of all original bands. The first set of second-order statistical bands is generated based on auto-correlation. They are constructed by multiplying each individual band by itself, i.e., \( \{ B_i B_i^\top \}_{i=1}^p \). A second set of second-order statistical bands are made up of all cross-correlated bands which are produced by correlating any pair of two different bands, i.e., \( \{ B_i B_j^\top \}_{i,j=1}^p \). If more bands are needed, nonlinear functions may be used to generate so called nonlinear correlated bands. For example, we may use a square-root function to produce \( \{ \sqrt{B_i} \}_{i=1}^p \), or a logarithm function to produce \( \{ \log B_i \}_{i=1}^p \) to stretch out lower gray level values. In what follows, we describe several ways to generate second-order correlated and nonlinear correlated bands.

It is worth noting that all the bands generated by the BGP are produced nonlinearly. These bands should offer useful information for classification because the classifier to be used is linear and linearly generated bands will not provide useful information to help the classifier improve performance.

V. EXPERIMENTAL RESULTS

The SPOT (Satellite Pour l’Observation de la Terra) data will be used for this experiment. It was collected in three spectral bands, two of which are from the visible region of electromagnetic spectrum referred to as band 1 (0.5-0.59 \( \mu m \) ) and band 2 (0.61-0.68 \( \mu m \) ), and the third band is from the near infrared region of electromagnetic spectrum referred to as band 3 (0.79-0.89 \( \mu m \) ). The ground sampling distance (GSD) is 20 meters. The three SPOT bands shown in Figure 1 were taken over Northern Virginia where there are the Falls Church High School, the Little River Turnpike, a lake in the upper right of the image and the Mill Creek Park.

There are four targets of interests in this image: building, roads, vegetation and water. In Fig. 2, the spectra of those targets are shown. In the first experiment, the error analysis for nonlinear function is conducted with linear mixture assumption. Without losing generality, building, roads and vegetation are linear mixed with abundance sum to one and nonnegative. The errors are calculated with auto-correlation, cross-correlation, square root and square root of cross-
correlation functions and shown in Fig. 3(a), (b), (c), (d), and the maximum errors are 7.9, 7.1, 5.6 and 3.6 respectively.

As shown in the Fig. 4, different nonlinear functions yield different classification results. With the four choices of nonlinear functions, the square root of cross-correlation yields the best contrast in road and vegetation, while autocorrelation performs best in classifying buildings. The classification result of cross-correlation performs the worst, which barely detects road and missing the water. Since water has low reflection in all these three bands, it is often difficult to detect it. Further error analysis for water may provide more information for the criterion to select the optimal nonlinear function set.

V. CONCLUSION

The GOSP is designed to deal with the situation when the number of bands is less than the number of the targets. It nonlinearly generates additional bands to relax the band number constraint, but it also introduces more errors. The experimental results showed that those nonlinear functions perform differently. But from this experiment, we can not find strong evidence to link the errors of nonlinear functions to linear mixture model and the classification performance. In this case, further investigations are required.

REFERENCES